

Question 12.

$$\text{Solve } \frac{x}{x-6} < 4$$

Solution

re-writing in std. form: $\frac{-3x+24}{x-6} < 0$

$$=P \quad \frac{-3(x-8)}{x-6} < 0$$

$$\frac{3(x-8)}{x-6} > 0 ,$$

$$\frac{3(x-8)}{x-6} > \frac{0}{3}$$

$$\Rightarrow \frac{x-8}{x-6} > 0 \Rightarrow x < 6 \text{ or } x > 8$$

$$\Rightarrow (-\infty, 6) \cup (8, \infty)$$

Question 10

Polynomial inequality:

$$2x^2 + 7x - 4 \leq 0$$

Solution

$2x^2 + 7x - 4$ can be factored to:

$$(2x-1)(x+4)$$

$$\Rightarrow (2x-1)(x+4) \leq 0$$

$$\Rightarrow 2x-1 \leq 0, \quad x+4 \leq 0$$

$$\Rightarrow -4 \leq x \leq \frac{1}{2}$$

$$= \left[-4, \frac{1}{2} \right]$$

Question 11

$$(x+3)(x-3)^2 > 0$$

Solution:

$$x+3 > 0, \quad (x-3)(x-3) > 0$$

$$x+3 > 0, \quad (x-3) > 0$$

$$\Rightarrow -3 < x < 3 \text{ or } x > 3$$

$$= (-3, 3) \cup (3, \infty)$$

Question 9:

Graph: $f(x) = \frac{2x}{x-2}$

Solution

Asymptotes:

(i) Horizontal : $\lim_{x \rightarrow \infty} \left(\frac{2x}{x-2} \right) = \lim_{x \rightarrow -\infty} \left(\frac{2x}{x-2} \right) = 2$

$\Rightarrow \text{H.A} = 2 \quad y = 2$

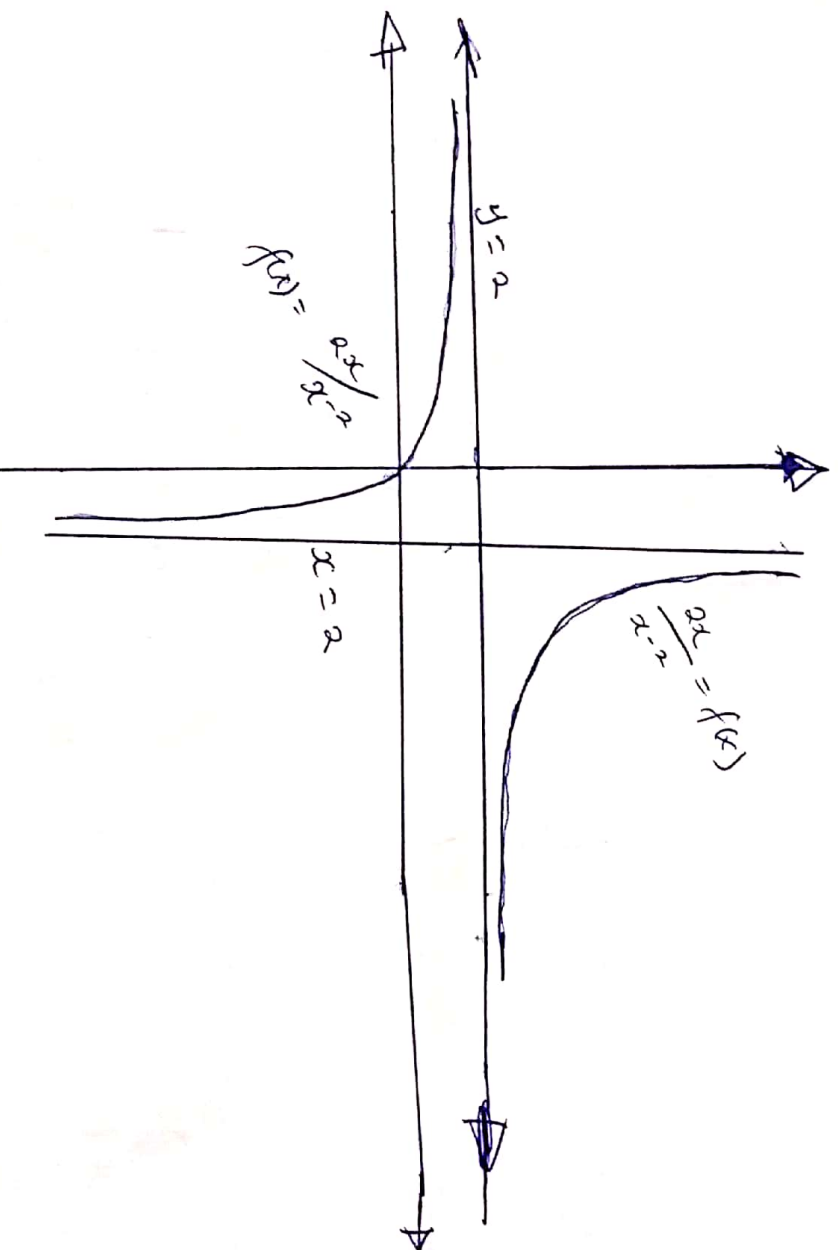
(ii) Vertical

$\frac{2x}{x-2}$ - square denominator with 0.

$$x-2=0 \Rightarrow x=2$$

(iii) X-intercept: $\frac{2x}{x-2} = 0 \Rightarrow x=0 \Rightarrow (0,0)$

Y-intercept, $\frac{2f(0)}{(0)-2} = y \Rightarrow y=0, (0,0)$



Question 6 Vertical Asymptotes:

$$q(x) = \frac{x+9}{x^2+9}$$

solution:

Equate denominator to 0.

$$\Rightarrow x^2 + 9 = 0$$

$$x^2 = -9$$

$$\Rightarrow x = -3i, 3i$$

$\Rightarrow$ No Vertical Asymptotes.

Question 7.

$$f(x) = \frac{6x}{3x^2+1}, \text{ horizontal Asymptote}$$

Solution

Put numerator = 0

$$\Rightarrow 6x = 0 \Rightarrow 6x = 0$$

$\therefore$ Degree of numerator < degree of denominator

$\therefore y = 0$ is the H.A.

=====

Question 8

$$f(x) = \frac{x^2 + 6x + 5}{x - x^2}$$

Solution

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 6x + 5}{x - x^2} \right) = -1$$

$$\lim_{x \rightarrow -\infty} \left(\frac{x^2 + 6x + 5}{x - x^2} \right) = -1$$

$y = L$ is horizontal Asymptote if
either $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$, and
 L is finite

$$\Rightarrow \text{H.A.} = y = -1$$

=====

Question 4

$$P(x) = -0.0002x^2 + 1.6x - 100, \quad x = \text{no. of pretzels}$$

Solution

The function is of the form: $ax^2 + bx + c = 0$

$$a = -0.0002, \quad b = 1.6, \quad c = -100$$

The no. of pretzels to maximize profit will be:

$$x = \frac{-b}{2a}$$

$$= \frac{-1.6}{2(-0.0002)} = \underline{\underline{400}}$$

$\therefore$ The number of pretzels that must be sold to maximize profit is 400

Question (5)

$$\frac{x^2 - 36}{x^2 - 9x + 14}, \quad \text{Vertical Asymptotes}$$

Solution

Put denominator $= 0$:

$$= 0 \quad x^2 - 9x + 14 = 0$$

$$\frac{9 \pm \sqrt{81 - 4(1)(14)}}{2(1)}$$

$$= 0 \quad x = 7 \quad \text{or } x = 2$$

Vertical Asymptotes

$$x = 7$$

$$\text{and } x = 2$$

Question 3.

Cost $C(x) = 2x^2 - 18x + 630$, x - no. of video rental.

Solution

$C(x)$ will be minimum if $\frac{d^2}{dx^2}(C(x)) > 0$.

$$\frac{d}{dx}(C(x)) = \underline{\underline{4x - 18}}, \quad \frac{d^2}{dx^2}(C(x)) = \underline{\underline{4}}$$

$$4x - 18 = 0$$

$$\Rightarrow 4x = 18$$

$$\Rightarrow x = \frac{9}{2}$$

$\Rightarrow C(x)$ is minimum at $x = \frac{9}{2}$

$$C\left(\frac{9}{2}\right) = 2\left(\frac{9}{2}\right)^2 - 18\left(\frac{9}{2}\right) + 630$$

$$= 589.5 \approx 590$$

$\Rightarrow$ Lowest cost $\approx \underline{\underline{590}}$ dollars

Question 2

Graph $y = -4(x+3)^2 - 3$

workings

i) Vertex

$$x_v = -\frac{b}{2a} = -\frac{-24}{2(-4)} = -3$$

$$y = -4((-3)+3)^2 - 3 = -3 \quad \left. \vphantom{\begin{matrix} y = -4((-3)+3)^2 - 3 = -3 \\ x_v = -\frac{b}{2a} = -3 \end{matrix}} \right\} \text{Vertex} = \text{maximum} \\ (-3, -3)$$

ii) Symmetry

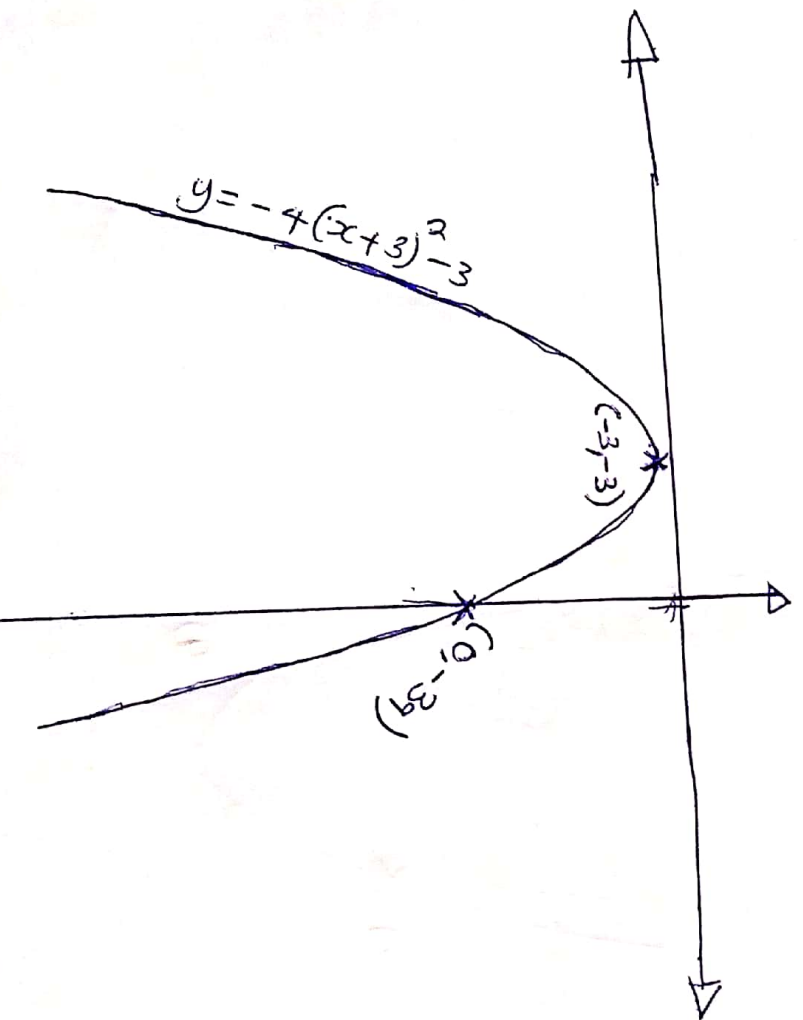
Axis of symmetry $\Rightarrow y = -\frac{b}{2a}$

$$= \frac{-(-24)}{2(-4)} \Rightarrow y = -3$$

(iii) Intercepts

y-intercept: $y = -4(0+3)^2 - 3 = -39$

x-intercept $\Rightarrow$ No x-intercept.



Question 1

1. $g(x) = x^2 - 4x + 8$

(i) Vertex

$$x_v = -\frac{b}{2a}, \quad a=1, \quad b=-4, \quad c=8$$

$$x_v = \frac{-(-4)}{2 \times 1} = 2$$

$$y_v = (2)^2 - 4(2) + 8 = 4$$

$\Rightarrow$ Vertex = minimum
(2, 4)

(ii) y-intercept

$$\left. \begin{aligned} g(x) &= 0^2 - 4(0) + 8 \\ &= 8 \end{aligned} \right\} \text{y-intercept} = (0, 8)$$

(iii) x-intercept

$$\left. \begin{aligned} 0 &= x^2 - 4x + 8 \\ \Rightarrow x &= \pm 2 \pm 2i \end{aligned} \right\} \text{no x-intercept}$$

